PHYSICS 2DL - SPRING 2010

MODERN PHYSICS LABORATORY

Monday May 17, 2010

Course Week 8

(2 LABs Left!)

Prof. Brian Keating

UCSD

UCSD

Final Exam

Thursday June 10 : 11:30 a.m. – 12:30 p.m

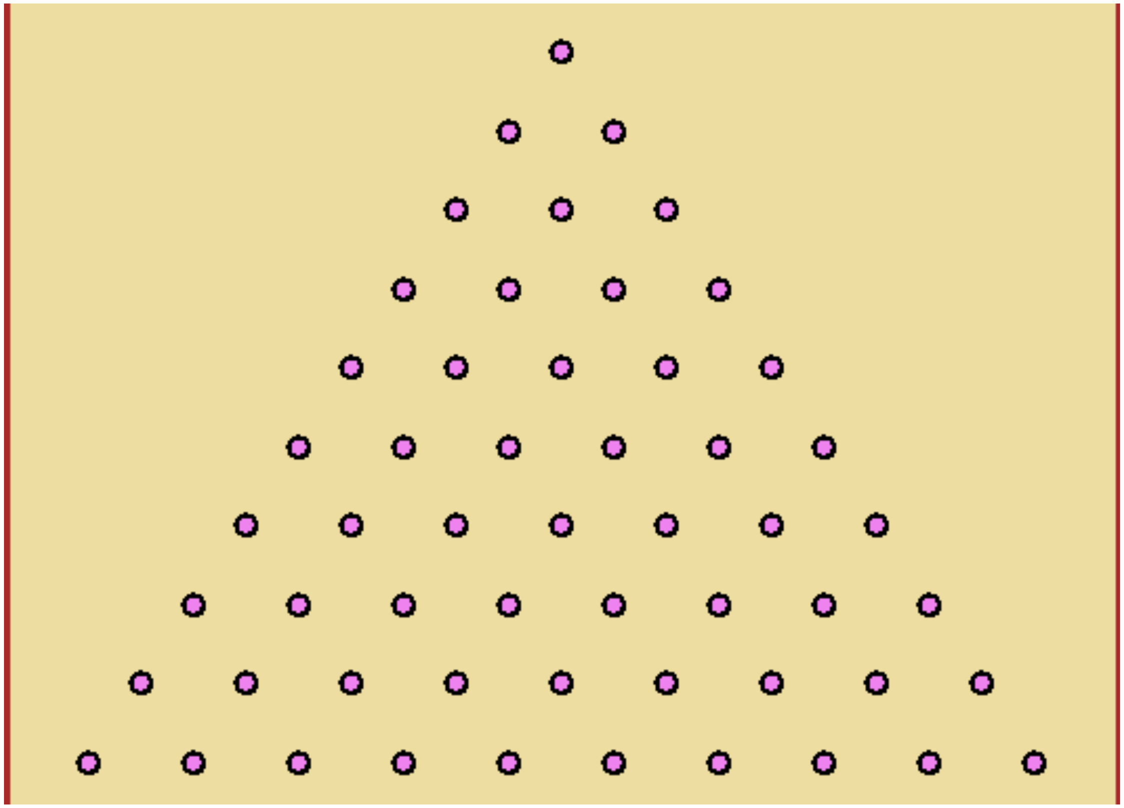
In classroom, Peterson 102

Covers all of Taylor that was covered in cHW

Covers statistical aspects of several experiments

Review in Class Next Monday (last class of quarter)

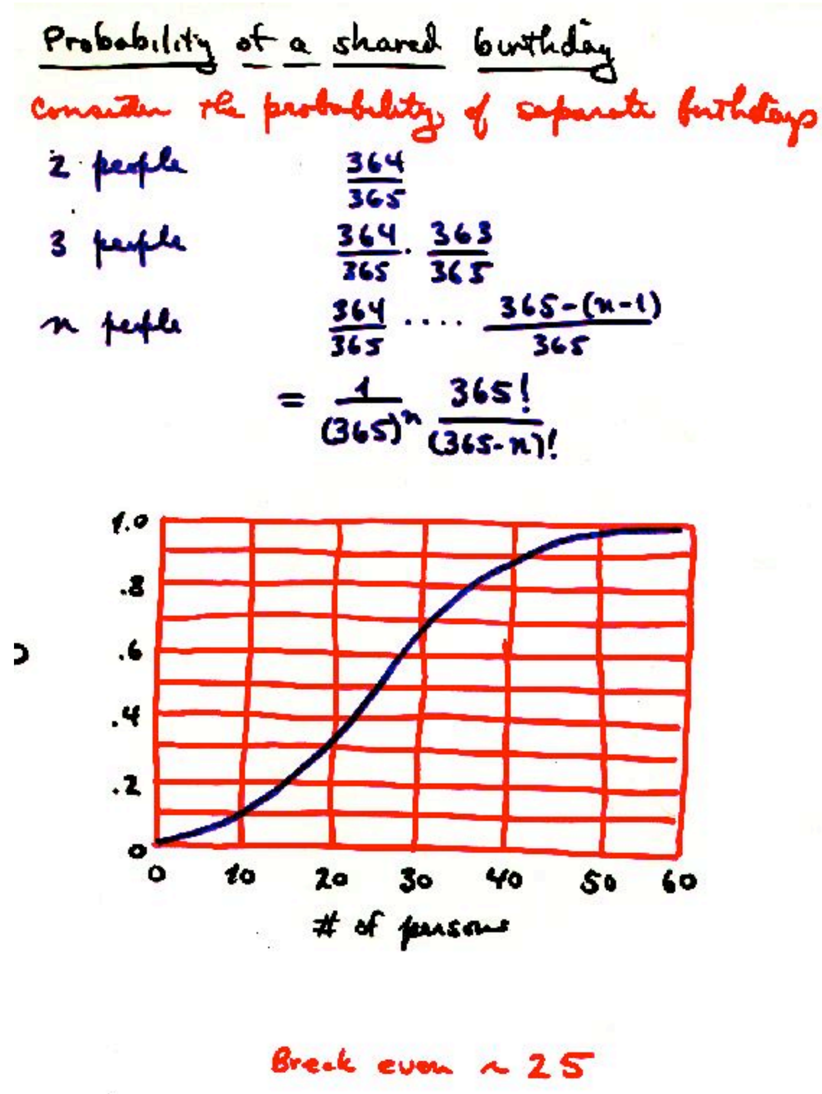
Ch 10 Binomial Distribution



Ch 10 Binomial Distribution

$$\mu_X = np$$

$$\sigma_X^2 = np(1-p)$$



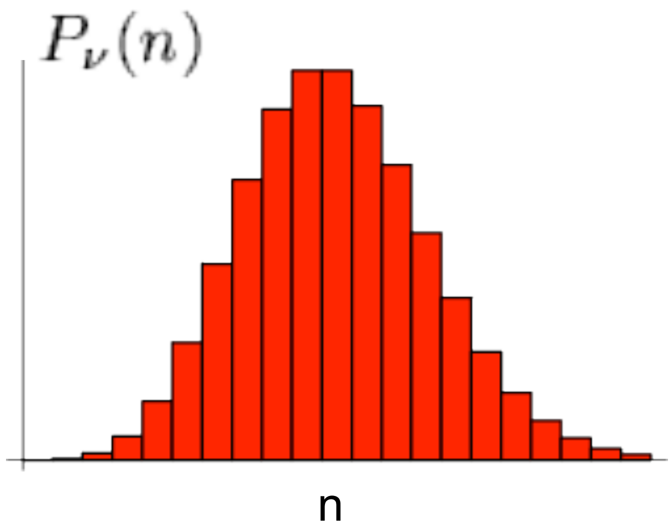
Today Ch 10 and Ch 11

- Review ch 8 least sq fit
- Ch 10 = Binomial Dist.
- Ch 11 = Poisson

Ch 11 Poisson Distribution

Given a Poisson process, the probability of obtaining exactly n successes in N trials is given by the limit of a binomial distribution

$$P_p(n|N) = \frac{N!}{n!(N-n)!}p^n(1-p)^{N-n}.$$



Now, instead of looking at getting n out of N if we define the number of of successes, we can set:

$$\nu \equiv Np$$

$$P_{\nu/N}(n \mid N) = \frac{N!}{n!\,(N-n)!}\left(\frac{\nu}{N}\right)^n \left(1-\frac{\nu}{N}\right)^{N-n}.$$

$$P_\nu(n) = \lim_{N\to\infty} P_p(n \mid N)$$

$$= \lim_{N\to\infty} \frac{N(N-1)\cdots(N-n+1)}{n!} \frac{\nu^n}{N^n} \left(1-\frac{\nu}{N}\right)^N \left(1-\frac{\nu}{N}\right)^{-n}$$

$$= \lim_{N\to\infty} \frac{N(N-1)\cdots(N-n+1)}{N^n} \frac{\nu^n}{n!} \left(1-\frac{\nu}{N}\right)^N \left(1-\frac{\nu}{N}\right)^{-n}$$

$$= 1 \cdot \frac{\nu^n}{n!} \cdot e^{-\nu} \cdot 1$$

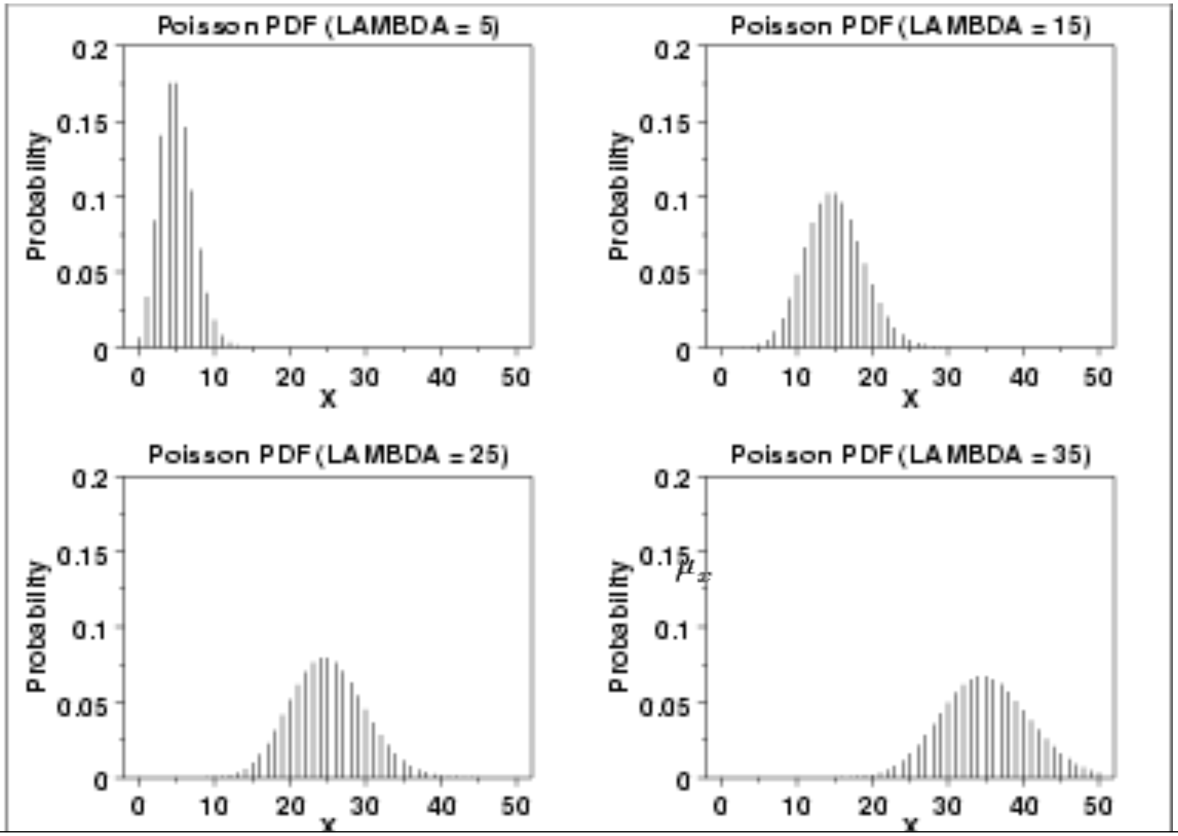
$$P_\nu(n) = \frac{\nu^n e^{-\nu}}{n!},$$

Ch 11 Poisson Distribution

$$P_\nu(n) = \frac{\nu^n e^{-\nu}}{n!},$$

Fixing p, when the sample size N become large, the distribution approaches a Gaussian.

Poisson formula
P(observing X occurrences)



Poisson – again should know when to use Gaussian

- Because Poisson is hard to calculate (with factorial) and because it's easier to find probability tabulated for Gaussian distribution.
- Approximation to use:

$$\overline{X} = \mu$$

$$\sigma = \sqrt{\mu}$$

- Then use t-values to answer likelihood questions.
- Caveat- N must be large! Better if large and symmetric.

UCSD's Budget Crisis!

- UCSD's budget crisis has reached epic proportions. They threaten to repo the new Triton statue near the Price Center. In January 2010 the Administration decided to save money by randomly cutting electrical power to the 2DL lab for 2 quarters...
- If electrical power failures occur according to a Poisson distribution with an average of 3 failures every twenty weeks, calculate the probability that the power will fail this week as you are doing the Franck-Hertz Experiment....

"Not more than one failure" means we need to include the probabilities for "0 failures" plus "1 failure".

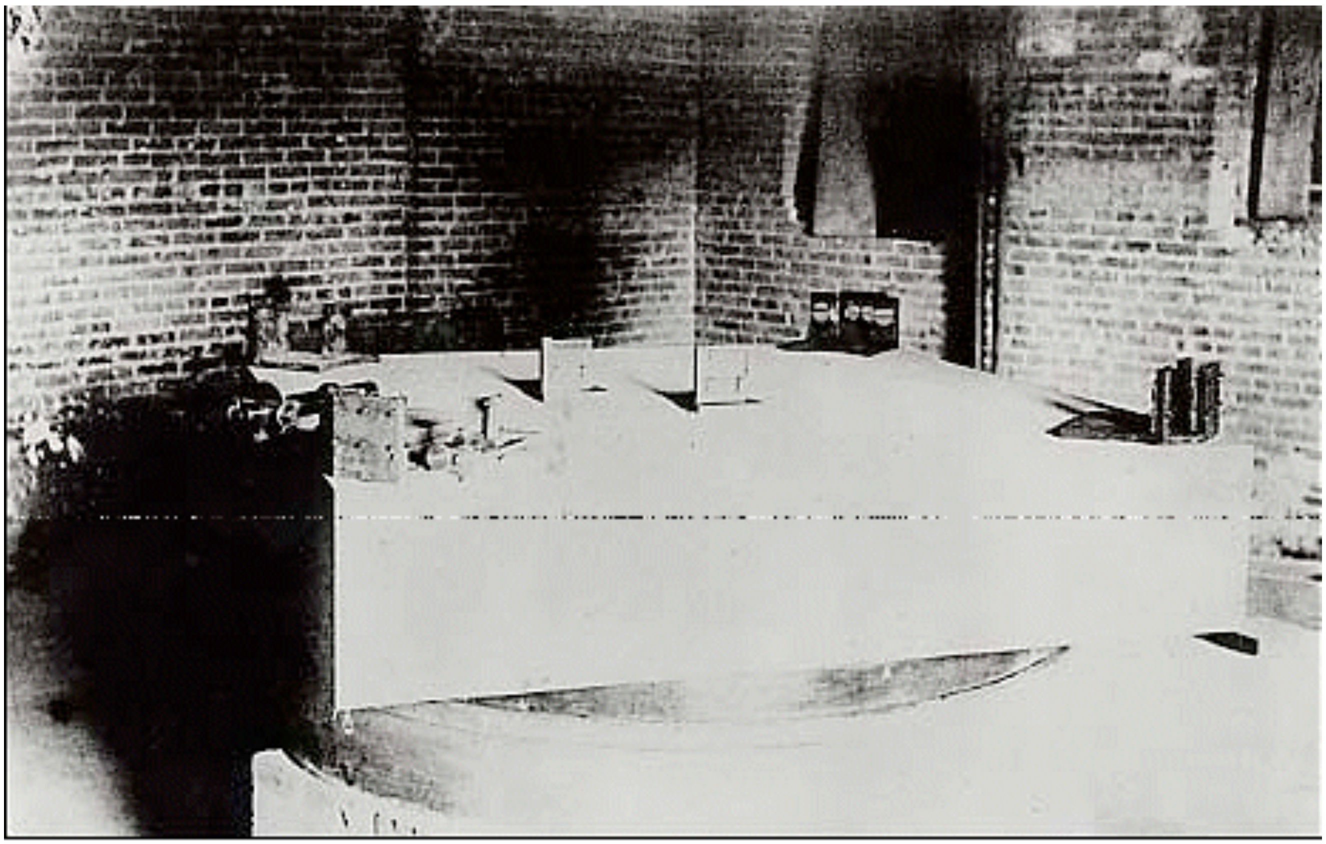
The average number of failures per week is: $\mu = \frac{3}{20} = 0.15$

"Not more than one failure" means we need to include the probabilities for "0 failures" & "1 failure".

$$P(x_0) + P(x_1) = \frac{e^{-0.15}0.15^0}{0!} + \frac{e^{-0.15}0.15^1}{1!} = 0.98981$$

$P_{screwed} = 1.019\%$

Optical Coherence & Interference



Michelson & Morley's 1887 interferometer
built in the basement of Western Reserve
Photo: Case Western Reserve Archive

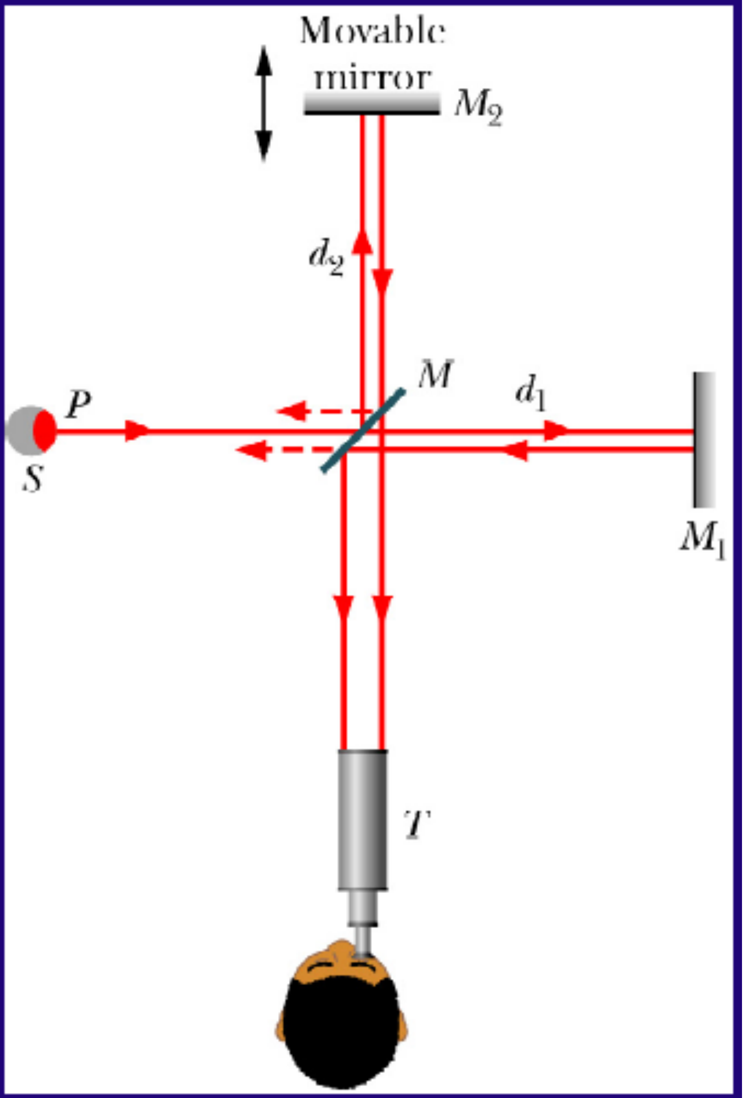
Michelson's Interferometer

Interferometer: device to measure lengths or changes in lengths with great accuracy by means of interference fringes (big daddy of them all was designed by Michelson in 1881…first American Nobel prize 1907)

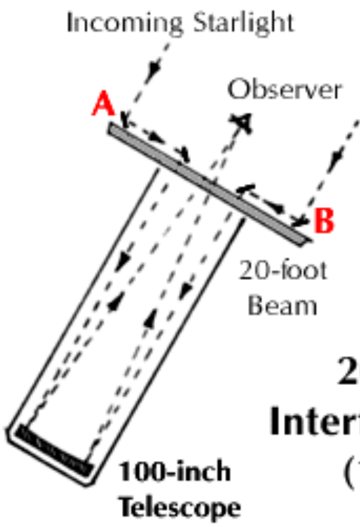
How it works:

- Light from source at P encounters beam splitter
- Beam splitter transmits ½ and reflects ½ of incident
- The 2 waves now head towards M1 and M2 mirrors
- Get reflected entirely and sent back along direction of incidence and then deflected towards telescope T
- Observer at T sees a pattern of "zebra strip" like fringes

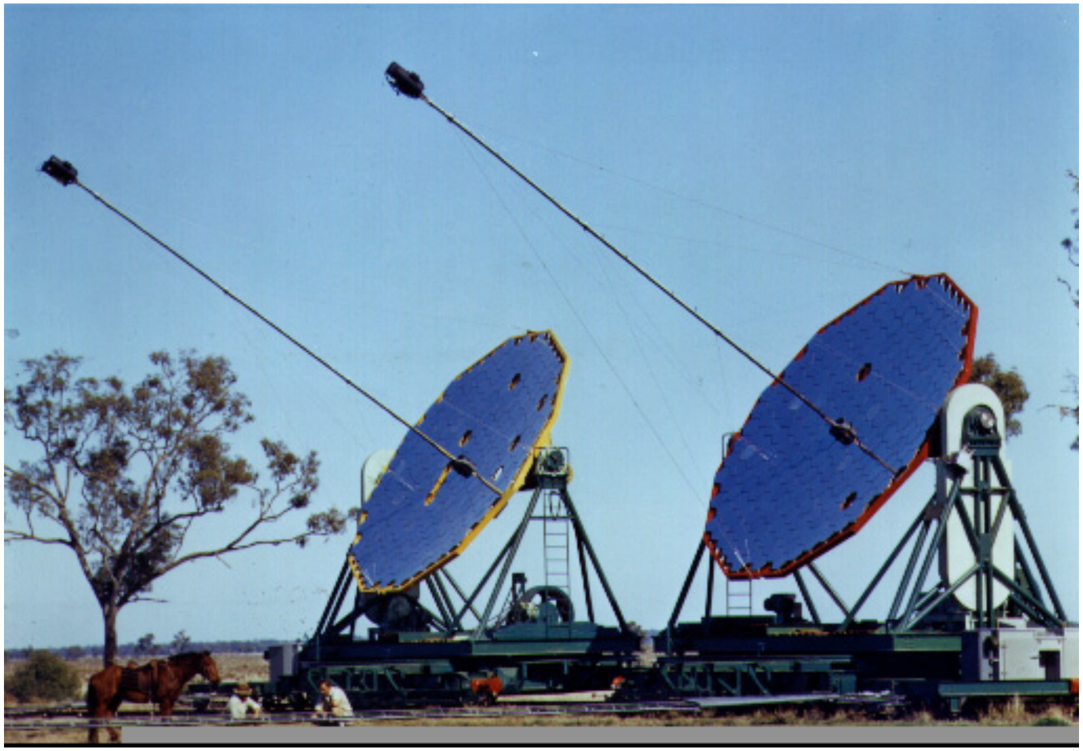
Path length Δ when 2 waves combine at telescope$=2d_2 - 2d_1$ anything that changes this path diff Δ will cause change in phase diff between two waves at the eye. E.g. If mirror M1 moves by $\lambda/2$ then Δ changes by λ and fringe pattern shifted by 1 (max→ min)



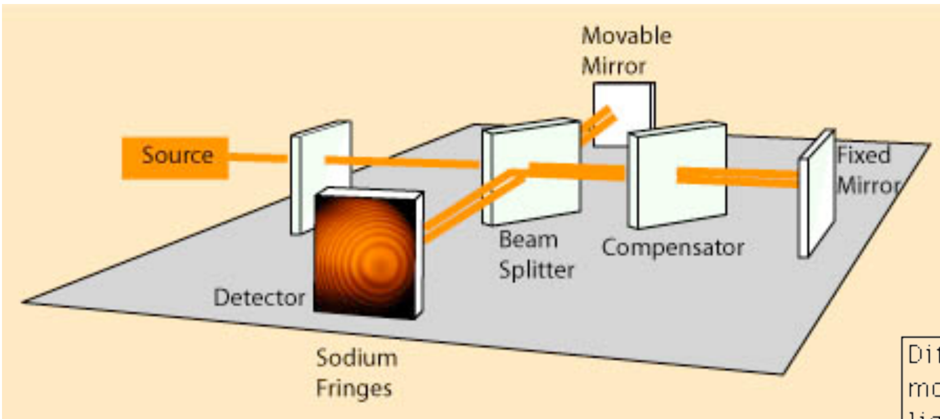
Stellar Interferometry

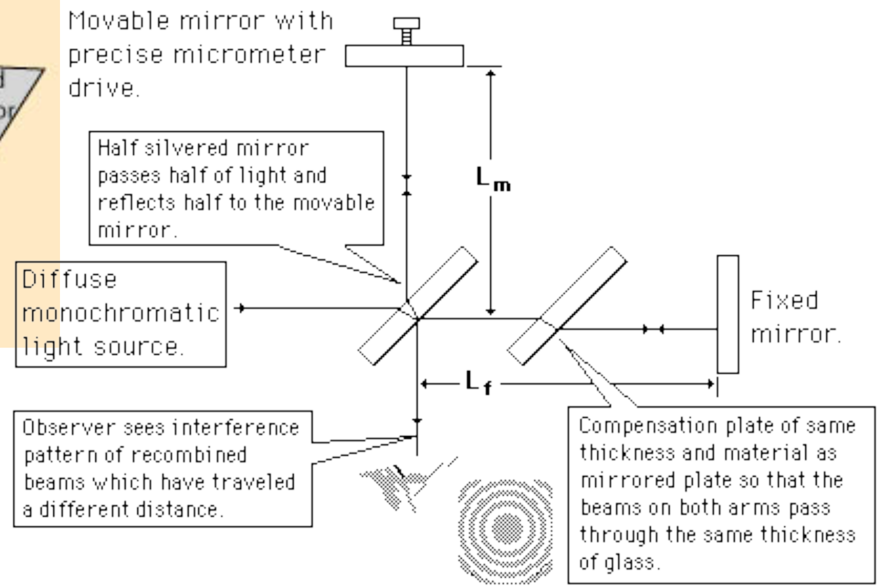


20-foot Interferometer (1919)



Interferometry





$$\ell_{coherence} = c \cdot t_{coherence}$$

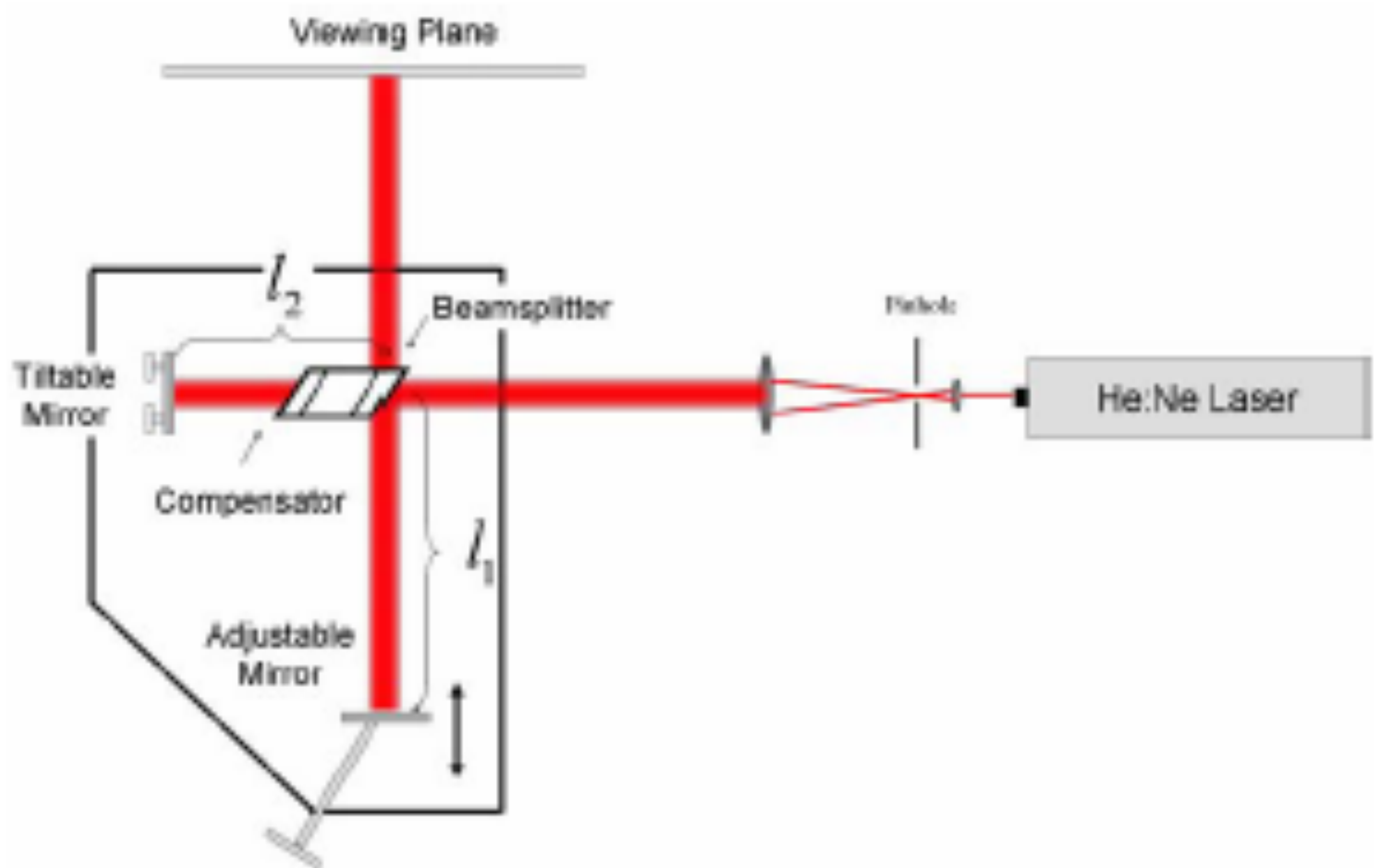
$$\begin{aligned}
\Delta\nu &= \nu_{\max} - \nu_{\min} \\
&= \frac{c}{\lambda_{\min}} - \frac{c}{\lambda_{\max}} = c \cdot \left(\frac{1}{\lambda_{\min}} - \frac{1}{\lambda_{\max}}\right) \\
&= c \cdot \left(\frac{\lambda_{\max} - \lambda_{\min}}{\lambda_{\max} \cdot \lambda_{\min}}\right) \\
&= \frac{c \cdot \Delta\lambda}{\lambda_{\max} \cdot \lambda_{\min}} \left[\frac{\text{cycles}}{\text{second}} = \text{Hz}\right]
\end{aligned}$$

For a laser, $\lambda_{\max} \simeq \lambda_{\min} \simeq \lambda_0$ and $\Delta\lambda \simeq 0$, so the temporal bandwidth is very small:

$$\Delta\nu \simeq \frac{c \cdot 0}{\lambda_0^2} \to 0$$

The reciprocal of the temporal bandwidth has dimensions of time and is the *coherence time*.

$$\begin{aligned}
t_{coherence} &= \frac{1}{\Delta\nu} [\text{s}] \to \infty \text{ for laser} \\
\ell_{coherence} &= \frac{c}{\Delta\nu} [\text{m}] = \frac{\lambda_{\max} \cdot \lambda_{\min}}{\Delta\lambda} \to \infty \text{ for laser}
\end{aligned}$$

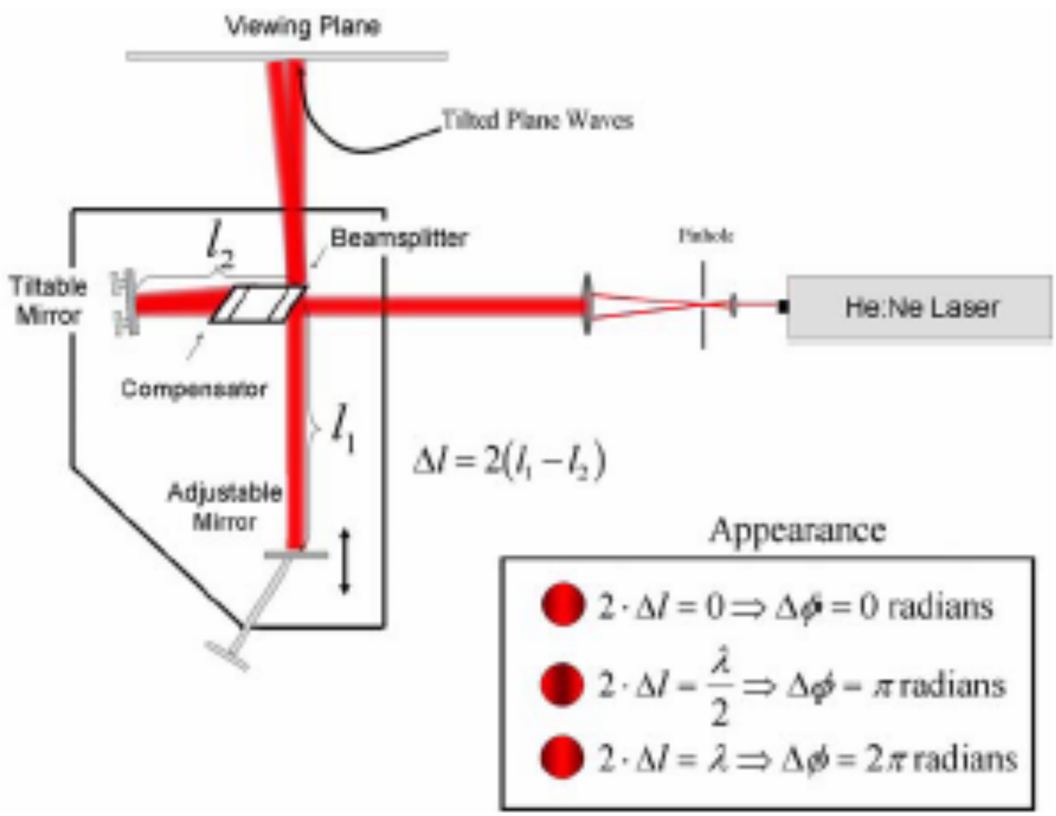


Michelson interferometer using He:Ne laser for illumination ($\lambda_0 = 632.8\,\text{nm}$).

$$OPD = \frac{2 \cdot \Delta \ell}{\lambda_0} \quad \text{[wavelengths]}$$

$$OPD = 2 \cdot \ell_1 - 2 \cdot \ell_2 = 2 \cdot \Delta \ell \quad \text{[m]}$$

$$\begin{aligned} O\Phi D &= 2\pi \left[\frac{\text{radians}}{\text{wavelength}}\right] \cdot \frac{2 \cdot \Delta \ell}{\lambda_0} \quad \text{[wavelengths]} \\ &= 2\pi \cdot \frac{2 \cdot \Delta \ell}{\lambda_0} \quad \text{[radians]} \end{aligned}$$



Michelson interferometer with collimated light so that one beam is tilted relative to the other. The optical phase difference varies linearly across the field, producing linear fringes just like Young's two-slit interference.